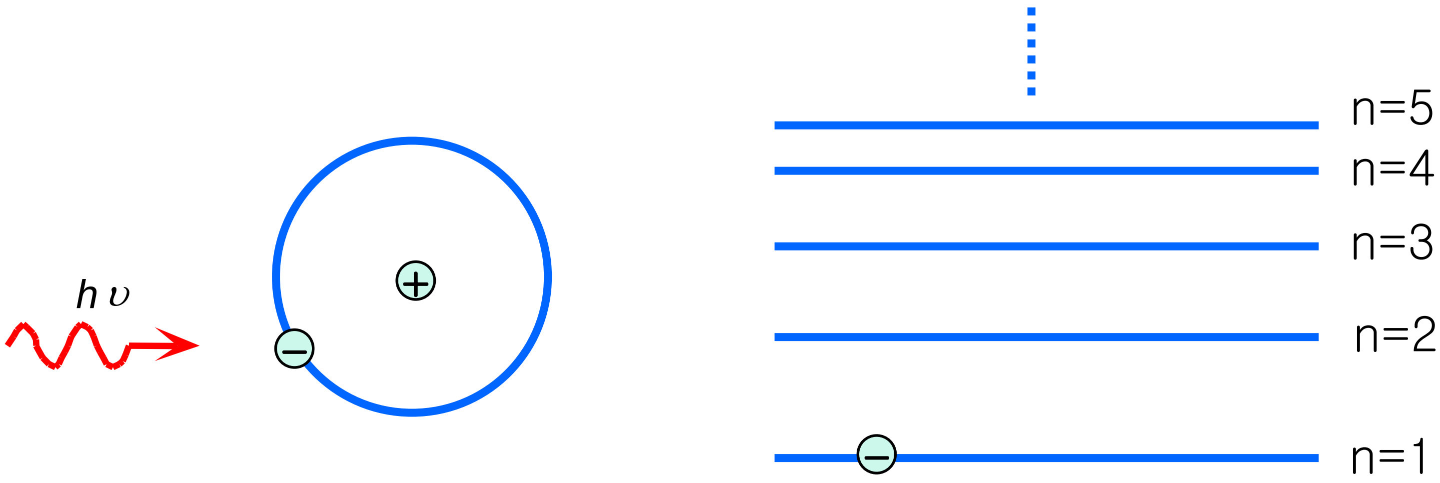
Lect. 16: Interaction between Light and Matter

What happens when photon energy matches energy transition levels in matter?

Electron energy levels in an hydrogen atom



$$E_n = -\frac{13.6}{n^2}[\text{eV}]$$

Energy levels inside every matter are quantized; details depend on the matter

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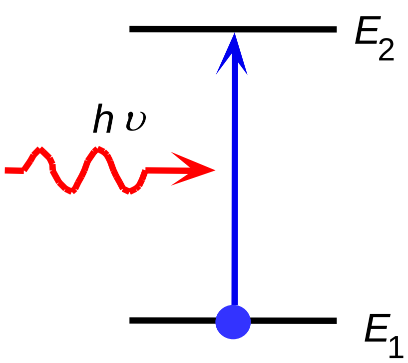
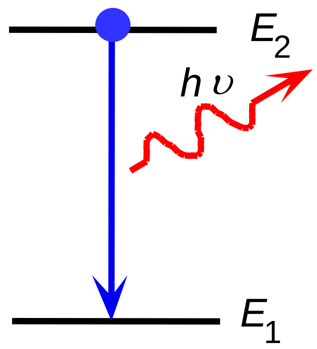
Consider only two energy levels: ground and excited states
Assume $h\nu = E_2 - E_1$

➔ Three interaction processes are possible

Absorption

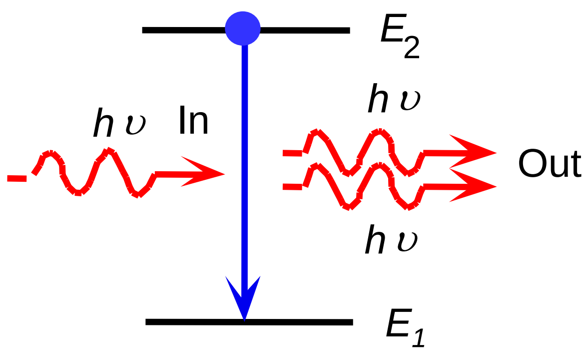


Spontaneous Emission



out photons are "random"

Stimulated Emission



out photons are "identical" to in photons: amplification

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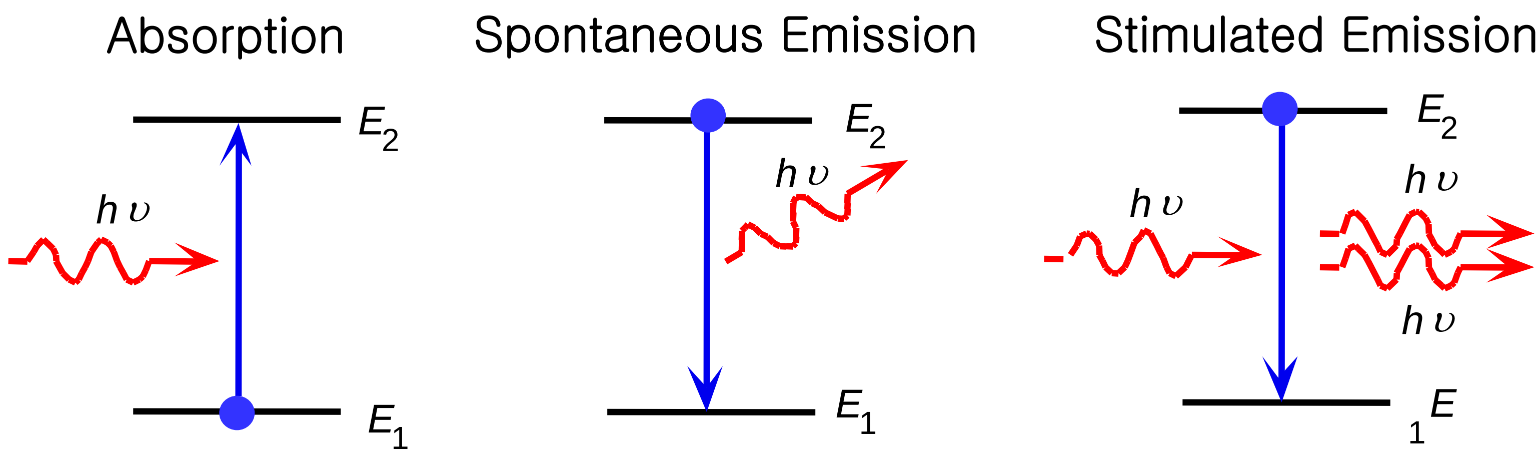
Determine the rate for each process



$$R_{12} = B_{12} \cdot N_1 \cdot \rho \qquad R_{sp} = A_{21} \cdot N_2 \qquad R_{21} = B_{21} \cdot N_2 \cdot \rho$$

ρ : photon density

$N_{1,2}$: electron density at $E_{1,2}$

B_{12}, B_{sp}, B_{21} : constants

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What is the dominant process at equalibrium (No net change of N_1, N_2, ρ)?

$$\therefore R_{12} = R_{sp} + R_{21}$$

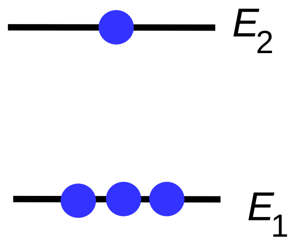
$$B_{12} \cdot N_1 \cdot \rho = A_{21} \cdot N_2 + B_{21} \cdot N_2 \cdot \rho$$

$$\rho = \frac{A_{21}/B_{12}}{\dfrac{N_1}{N_2} - \dfrac{B_{21}}{B_{12}}}$$

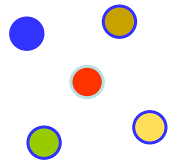
From another branch of physics (statistical mechanics)

Electron distribution at equalibrium

$$\frac{N_2}{N_1} = \exp\left(-\frac{E_2 - E_1}{kT}\right)$$



Photon distribution at equalibrium
(Planck law for black-body radiation)



$$\rho(h\upsilon) = \frac{8\pi h\upsilon^3}{c^3\left[\exp\left(\dfrac{h\upsilon}{kT}\right) - 1\right]}$$

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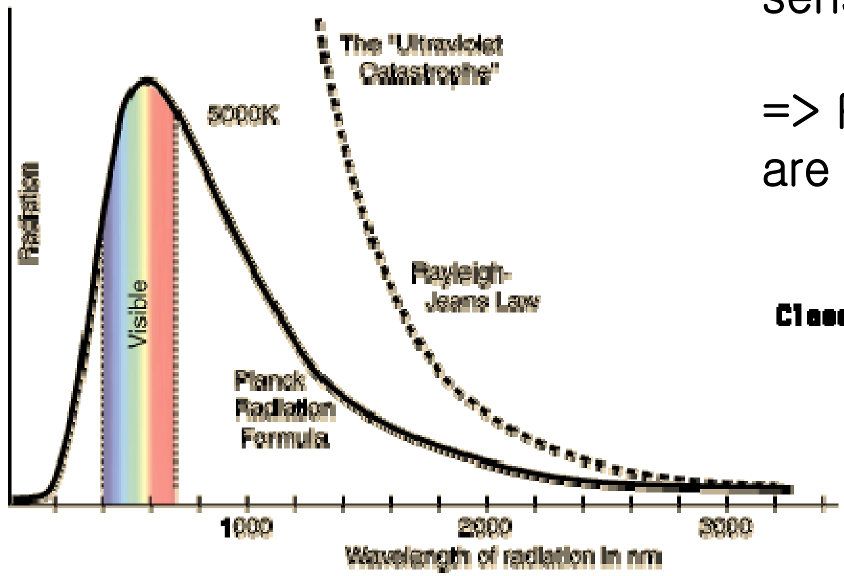
Consider the spectrum of light emission from an heated object (Thermal emission)
(heat => oscillatrion of charges inside the object => EM emission)

Very detailed analysis is possible for “black body” radiation (Rayleigh-Jeans Law).
(black body: object that absorbs 100% of incoming EM radiation => 100% emission)
=> Max. EM emission from a heated object at a given temperature)



Rayleigh-Jeans law did not make sense at high frequencies.

=> Planck suggested that EM energies are quantized (photon) $E_{photon} = h\nu$.

Classical — Rayleigh-Jeans Law

$$\frac{8\pi\nu^2}{c^3} kT$$

Planck Law — Quantum

$$\frac{8\pi\nu^2}{c^3} \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}$$

Planck Law approaches R-J Law when $h\nu \ll kT$.

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Compare

$$\rho(E_2 - E_1) = \frac{A_{21}/B_{12}}{e^{\left(\frac{E_2 - E_1}{kT}\right)} - \frac{B_{21}}{B_{12}}} \quad \text{and} \quad \rho(h\upsilon) = \frac{8\pi h\upsilon^3}{c^3\left[e^{\left(\frac{h\upsilon}{kT}\right)} - 1\right]}$$

Assuming hν = E_1 − E_2 , two expressions should be identical.

$$\frac{B_{21}}{B_{12}} = 1 \text{ and } \frac{A_{21}}{B_{12}} = \frac{A_{21}}{B_{21}} = \frac{8\pi h\upsilon^3}{c^3}$$

Interpretations:

- Absorption or simulated emission depends only on N_1 and N_2
- Spontaneous emission and stimulated emission are intrinsically related

➔ Einstein's A, B constants

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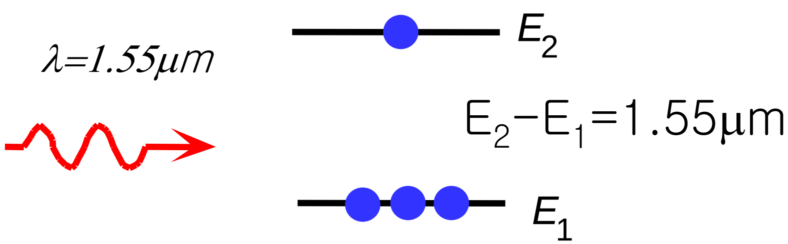
Which process is dominant at equalibrium?

Stimulated emission vs. absorption

$$\frac{R_{21}}{R_{12}} = \frac{B_{21}N_2\rho}{B_{12}N_1\rho} = \frac{N_2}{N_1} = \exp\left(-\frac{E_2 - E_1}{kT}\right) << 1$$



$$\frac{R_{21}}{R_{12}} = \frac{N_2}{N_1} = \exp\left(-\frac{0.8eV}{0.04eV}\right) \sim 2\times10^{-9}$$

Virtually no possibility for stimulated emission at equalibrium

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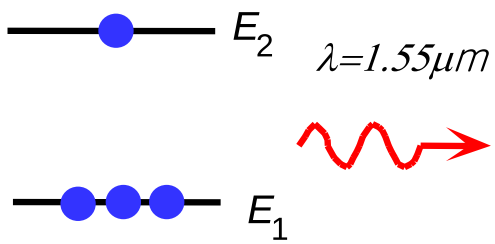
Which process is dominant at equalibrium?

Stimulated emission vs. spontaneous emission

$$\frac{R_{21}}{R_{sp}} = \frac{B_{21}N_2\rho}{A_{21}N_2} = \frac{B_{21}}{A_{21}}\bullet\rho = \frac{c^3}{8\pi h\upsilon^3}\bullet\frac{8\pi h\upsilon^3}{c^3\left[\exp\left(\frac{h\upsilon}{kT}\right)-1\right]} = \frac{1}{\exp\left(\frac{E_2-E_1}{kT}\right)-1}$$



$$\frac{R_{21}}{R_{sp}} = \frac{1}{\exp\left(\frac{0.8eV}{0.04eV}\right)-1} = \frac{1}{4.84\times10^8-1} \sim 2\times10^{-9}$$

Virtually all photon emission is due to spontaneous emission at equalibrium

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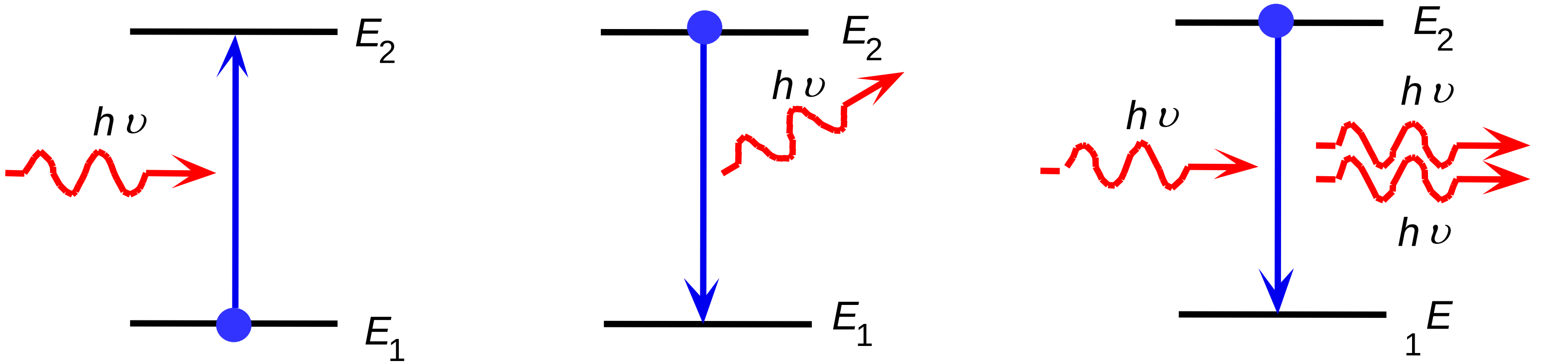
How can we cause stimulated emission?

Absorption | Spontaneous Emission | Stimulated Emission



$$R_{12} = B_{12} \cdot N_1 \cdot \rho \qquad R_{sp} = A_{21} \cdot N_2 \qquad R_{21} = B_{21} \cdot N_2 \cdot \rho$$

Make N_2 larger than N_1: Break equalibrium and "pump" carriers into E_2

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Exercise Problems:

Prob. 12 in Part 2, Prob. 1 in Part 3